Math 60 10.6 Polynomial Inequalities

Objectives 1) Solve quadratic inequalities
2) Solve polynomial inequalities

Note: All problems in this section are one-variable.

GOAL: (1) Find values of x that make $x^2-4x-5 \geq 0$ true.

Let's guess a little:

$x = -4$ $\quad (-4)^2-4(-4)-5 > 0?$
$27 > 0$ true, $x=-4$ is a solution

$x = -2$ $\quad (-2)^2-4(-2)-5 > 0?$
$7 > 0$ true, $x=-2$ is a solution

$x = 0$ $\quad 0^2-4(0)-5 > 0?$
$-5 > 0$ false, $x=0$ is not a solution

$x = 2$ $\quad 2^2-4(2)-5 > 0?$
$-9 > 0$ false, $x=2$ is not a solution

$x = 4$ $\quad 4^2-4(4)-5 > 0?$
$-5 > 0$ false, $x=4$ is not a solution

$x = 6$ $\quad 6^2-4(6)-5 > 0?$
$7 > 0$ true, $x=6$ is a solution

$x = 8$ $\quad 8^2-4(8)-5 > 0?$
$27 > 0$ true, $x=8$ is a Solution

We notice: There are many solutions.
We will not list them individually, so we'll be using interval notation.

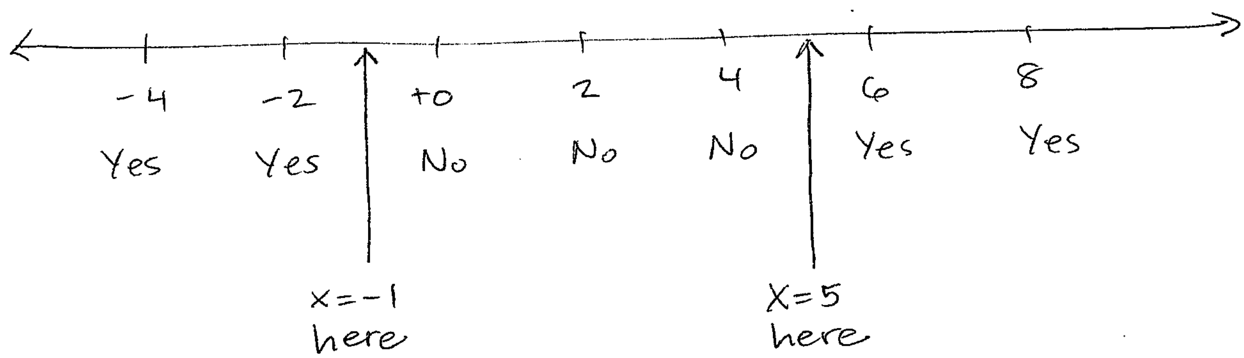
Math 60 10.6

Notice: For what values of x is $x^2-4x-5=0$?

$(x-5)(x+1)=0$

$x=5 \quad x=-1$

Where do $x=5$ and $x=-1$ fit in our list of guesses?



Notice that to the left of $x=-1$, every number we tested is a solution,

Similarly, between $x=-1$ and $x=5$, every number we tested is not a solution.

And to the right of $x=5$, every number we tested is again a solution.

The solution set for $x^2-4x-5 \geq 0$ can be written

as $\{x \mid x \leq -1 \text{ or } x \geq 5\}$

or in interval notation

$\boxed{(-\infty, -1] \cup [5, \infty)}$

Math 60. 10.6

Let's get a more efficient method, instead of guessing a lot of numbers.

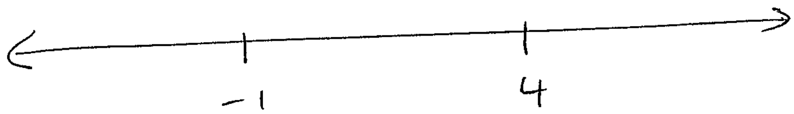
② Solve $x^2-3x-4 \geq 0$

step 1: Find the x-intercepts of the graph $f(x)=x^2-3x-4$ also called the solutions of $x^2-3x-4=0$.

Shortcut: Temporarily change the question to an equation by replacing $\geq$ by $=$.

$$x^2-3x-4=0$$
$$(x-4)(x+1)=0$$
$$x=4 \quad x=-1$$

step 2: Plot these solutions on a number line.



step 3: test one value of x for each region on the number line.

Left of -1 : test $x=-2$

$(-2)^2-3(-2)-4 \geq 0$?

$6 > 0$ yes. $\Rightarrow$ shade this area

Between -1 and 4: test $x=0$

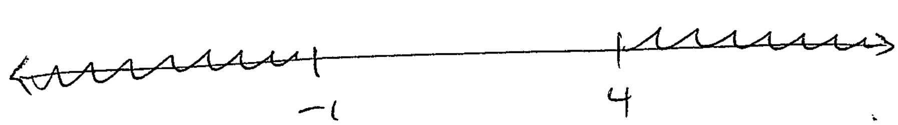
$0^2-3(0)-4 \geq 0$?

$-4 > 0$ no $\Rightarrow$ do not shade this area

Right of 4 : test $x=5$

$5^2-3(5)-4 \geq 0$?

$6 > 0$ yes $\Rightarrow$ shade this area

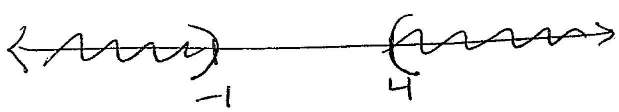


Math 60 10.6

step 4: Determine whether to use brackets [ ] or parentheses ( ).

Use [ ] if the question is $\le$ or $\ge$

Use ( ) if the question is $<$ or $>$.



step 5: Write the solution using interval notation.

$$\boxed{(-\infty, -1) \cup (4, \infty)}$$

(3) Solve $-x^2 + 8 > 2x$

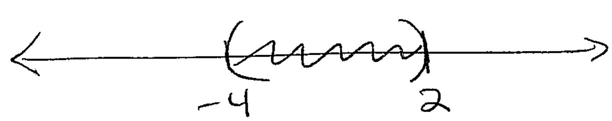
step 1: $-x^2 + 8 = 2x$

$0 = x^2 + 2x - 8$

$0 = (x+4)(x-2)$

$x = -4, 2$

step 2:



test $x = -5$ $\quad -(-5)^2 + 8 > 2(-5)$?

$-17 > -10$ false

test $x = 0$ $\quad -0^2 + 8 > 2(0)$

$8 > 0$ true

test $x = 3$ $\quad -3^2 + 8 > 2(3)$

$-1 > 6$ false

step 3: use ( )

step 4: $\boxed{(-4, 2)}$

Math 60 10.6

(4) Solve $2x^2 + 4x > 1$

Step 1: $2x^2 + 4x = 1$

$2x^2 + 4x - 1 = 0$

$D = b^2 - 4ac$

$= 4^2 - 4(2)(-1)$

$= 24$

not a perfect square, does not factor! But it does have two real solutions.

quadratic formula

$$x = \frac{-4 \pm \sqrt{24}}{2(2)}$$

$$= \frac{-4}{4} \pm \frac{2\sqrt{6}}{4}$$

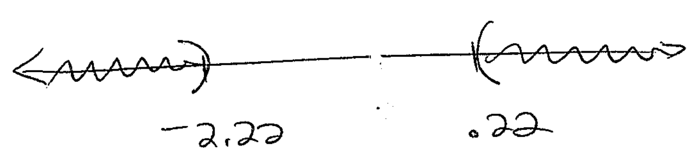
$$= -1 \pm \frac{\sqrt{6}}{2}$$

To put these on the number line, we need to know approximate values

$$-1 + \frac{\sqrt{6}}{2} \approx .22$$

$$-1 - \frac{\sqrt{6}}{2} \approx -2.22$$

Step 2



test $x = -3$ $\quad 2(-3)^2 + 4(-3) > 1$?

$6 > 1$ true

test $x = 0$ $\quad 2(0)^2 + 4(0) > 1$?

$0 > 1$ false

test $x = 1$ $\quad 2(1)^2 + 4(1) > 1$?

$6 > 1$ true

Use exact expressions in answer!

Step 3: parentheses

Step 4: $\boxed{\left(-\infty, -1 - \frac{\sqrt{6}}{2}\right) \cup \left(-1 + \frac{\sqrt{6}}{2}, \infty\right)}$

Math 60 10.6

(5) Solve $x^3 + x^2 - 9x - 9 > 0$

Step 1: $\underbrace{x^3 + x^2}_{} \underbrace{- 9x - 9}_{} = 0$

factor by grouping

GCF #1 x^2 GCF #2 -9

$x^2(x+1) - 9(x+1) = 0$

GCF #3 $(x+1)$

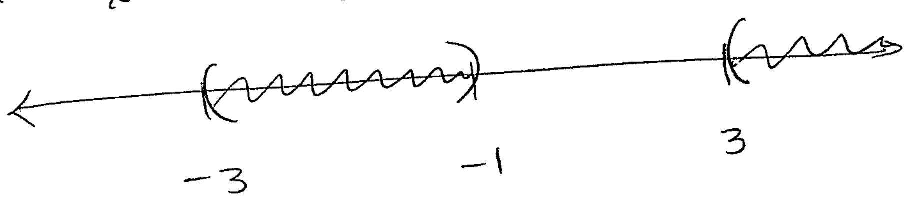
$(x+1)(x^2-9) = 0$

factor difference of squares

$(x+1)(x+3)(x-3) = 0$

$x = -1 \quad x = -3 \quad x = 3$

Step 2:



test $x = -4$ $\quad (-4)^3 + (-4)^2 - 9(-4) - 9 > 0$?

$-21 > 0$ no

test $x = -2$ $\quad (-2)^3 + (-2)^2 - 9(-2) - 9 > 0$?

$5 > 0$ yes

test $x = 2$ $\quad (2)^3 + (2)^2 - 9(2) - 9 > 0$?

$-15 > 0$ no

test $x = 4$ $\quad (4)^3 + (4)^2 - 9(4) - 9 > 0$?

$35 > 0$ yes.

Step 3: > means parentheses

Step 4: $\boxed{(-3,-1) \cup (3, \infty)}$

Math 60 10.6

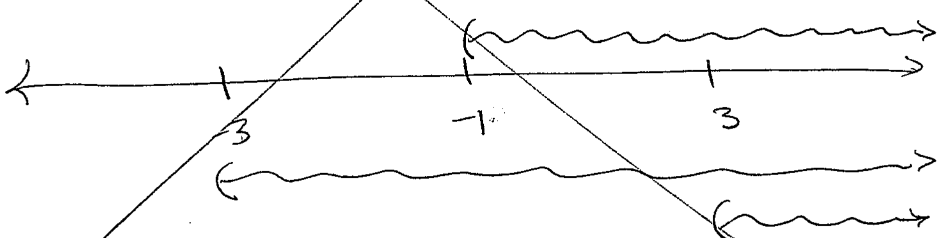
Common error: Trying to avoid sign chart and testing, a student says

$x^2 + x^2 - 9x - 9 > 0$

$(x+1)(x+3)(x-3) > 0$ as before

then $x+1 > 0$ $\quad x+3 > 0$ $\quad x-3 > 0$

$x > -1$ $\quad x > -3$ $\quad x > 3$



$x > -1$ shades between -1 & 3 which should not

So does $x > -3$

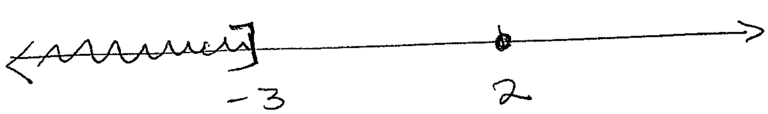
$x > 3$ misses the segment between -3 and -1.

So do not do this!

However: Some students notice that the shaded and unshaded regions alternate. This observation is often true ... but not if one factor is squared.

6 Solve $(x-2)^2(x+3) \le 0$

$x = 2 \qquad x = -3$



test $x = -4$ $\qquad (-4-2)^2(-4+3) \le 0$?

$-26 \le 0$ yes

test $x = 0$ $\qquad (0-2)^2(0+3) \le 0$

$12 \le 0$ no

test $x = 3$ $\qquad (3-2)^2(3+3) \le 0$

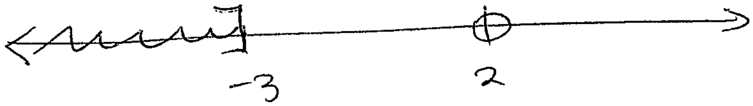
$6 \le 0$ no.

Math 60 10.6

Solution is an interval plus a single value

$$\boxed{(-\infty, -3] \cup \{2\}}$$

⑦ Solve $(x-2)^2(x+3) < 0$



not $= 0$, so 2 doesn't work

$$\boxed{(-\infty, -3]}$$